Using AdS/CFT for better Quantum Computing

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Introduction

- **3rd year undergraduate in physics and math**
- **Fun fact:** I once self-studied Mario Kart to beat everybody in my friend group
- **Fun fact:** I've eaten over 97 pieces of sushi in one sitting

Where I've been

Theoret[illegible] Low-Dimensio[illegible]
Superconductivity @ UIUC

Quantum Algorithms
@ Johns Hopkins APL

WARNING

THIS TALK IS NOT RIGOROUS

I just want to spark interest in this field

An Ambitious Agenda

1. The themes of AdS/CFT
2. What are Tensor Networks?
3. Holographic Tensor Networks
4. Holographic Quantum Algorithms

An Ambitious Agenda

1. **The themes of AdS/CFT**
2. What are Tensor Networks?
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A Brief Motivation into AdS/CFT

Entropy ~ Information of a system

1974: Bekenstein and Hawking derived

$$\text{Black Hole Entropy} \propto \text{Area}$$

A Brief Motivation into AdS/CFT

Black Hole Entropy $\propto$ Area

Information of entire system described by

"**holographic**" projection

A Question

Black holes aren't anything THAT special…

Q: Is there a relationship between Bulk Information and Boundary information?

A: Yes!

AdS/CFT

1997: Juan Maldacena proposed equivalence between 5 dim Anti-de Sitter spacetime and 4 dim supersymmetric Yangs-Millis Theory (Type IIB String Theory)

1.) Anti-de Sitter (AdS) Space: 1 of 3 solutions to GR equations

2.) 4-d Yangs-Millis Theory: A conformal field theory (CFT), invariant under rotation but not length preserving transformations

AdS/CFT

1997: Juan Maldacena ~~proposed equivalence between 5 dim Anti-de Sitter spacetime and 4 dim supersymmetric Yangs-Millis Theory (Type IIB String Theory)~~

AdS/CFT

1997: Juan Maldacena proposed the first instance of

Bulk Information ⟺ Boundary Information

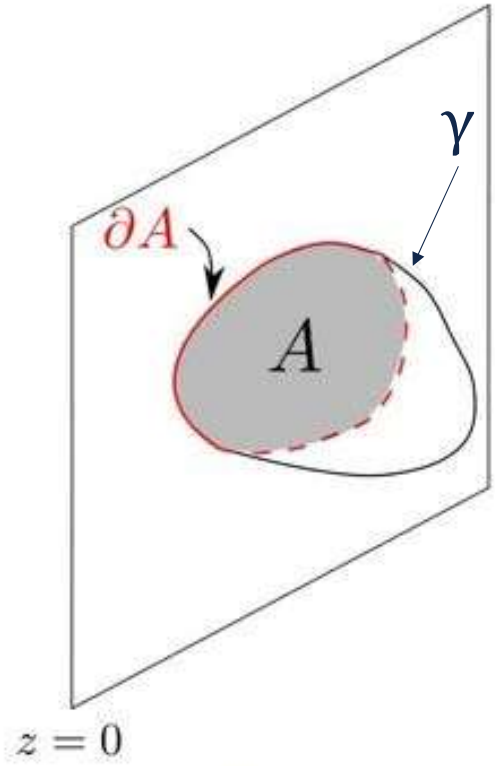
Called AdS/CFT, or the Holographic Principle

Connection to Quantum Information

2006: Ryu and Takayanagi proposed

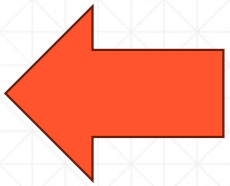
Entanglement Entropy of A ***in CFT***

$\propto$ Area of Curve (γ) ***in AdS***



First time AdS/CFT in quantum info

An Ambitious Agenda

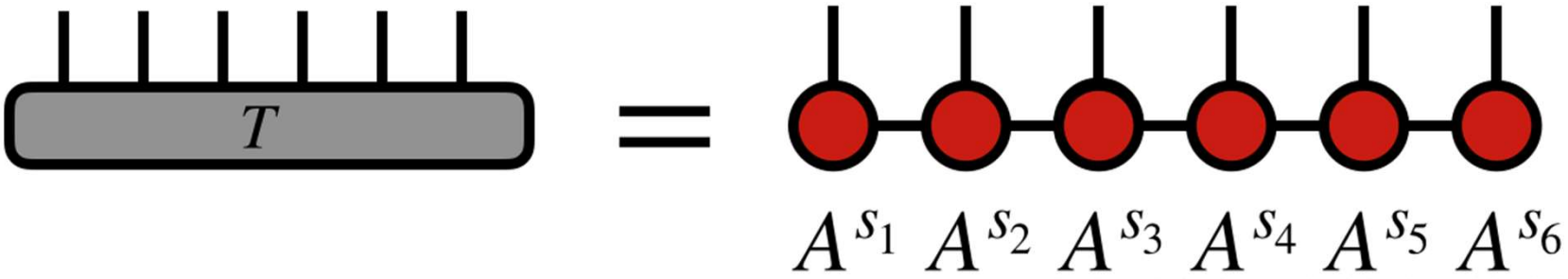
1. ~~The themes of AdS/CFT~~
2. **What are Tensor Networks?** 
3. Holographic Tensor Networks
4. Holographic Quantum Algorithms

A Brief Introduction into Tensors

$$\mathrm{T}_{i_1,i_2,\ldots,i_n}$$

$$= A^{i_1}_{\alpha_1} A^{i_2}_{\alpha_1,\alpha_2} \ldots A^{i_n}_{\alpha_{n-1},\alpha_n}$$

"1-D Tensor Decomposition" or "Matrix Product States"



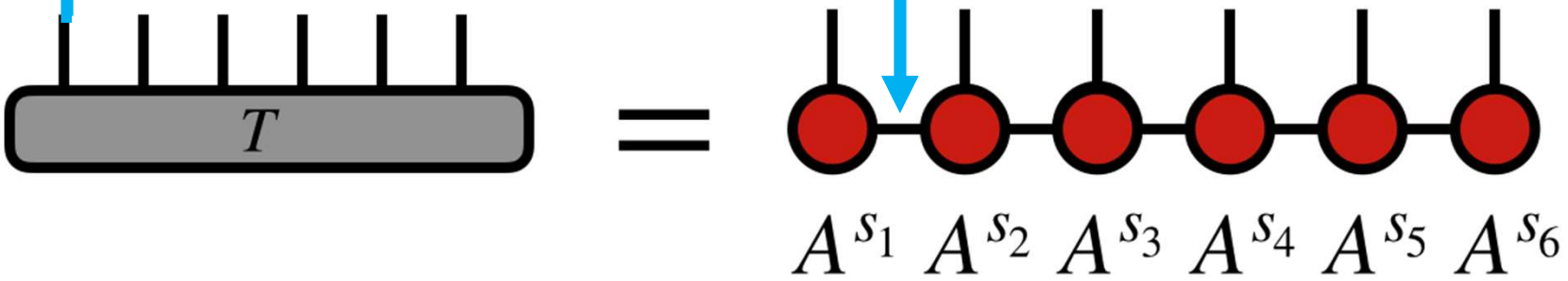
Also, a graph!

A Brief Introduction into Tensors

$$T_{i_1,i_2,\ldots,i_n} = A^{i_1}_{\alpha_1} A^{i_2}_{\alpha_1,\alpha_2} \ldots A^{i_n}_{\alpha_{n-1},\alpha_n}$$

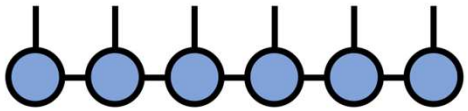
"Physical" Indices

"Contraction" or "Virtual" Indices

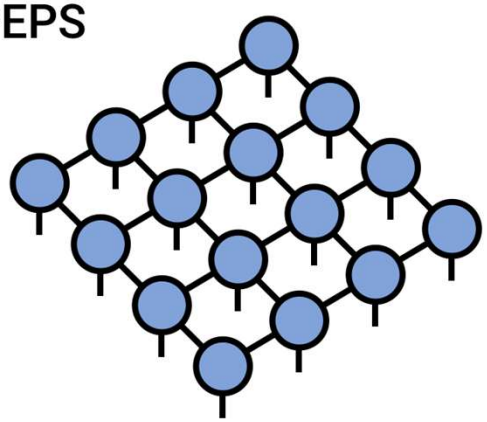


A Brief Introduction into Tensors

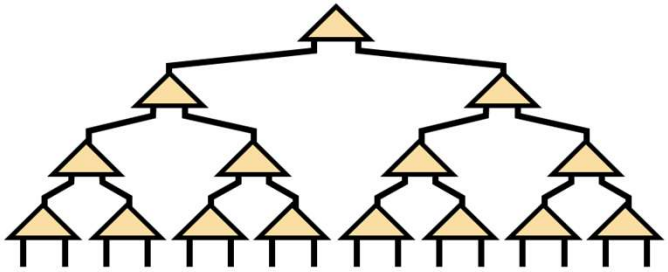
Matrix Product State /
Tensor Train



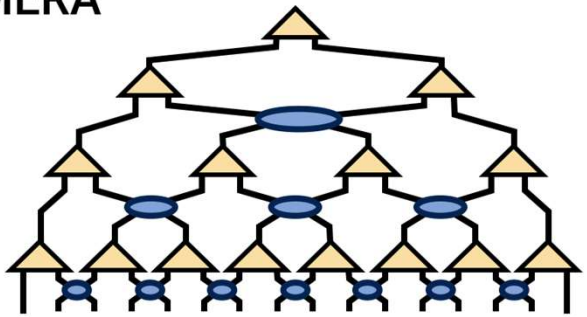
PEPS



Tree Tensor Network /
Hierarchical Tucker

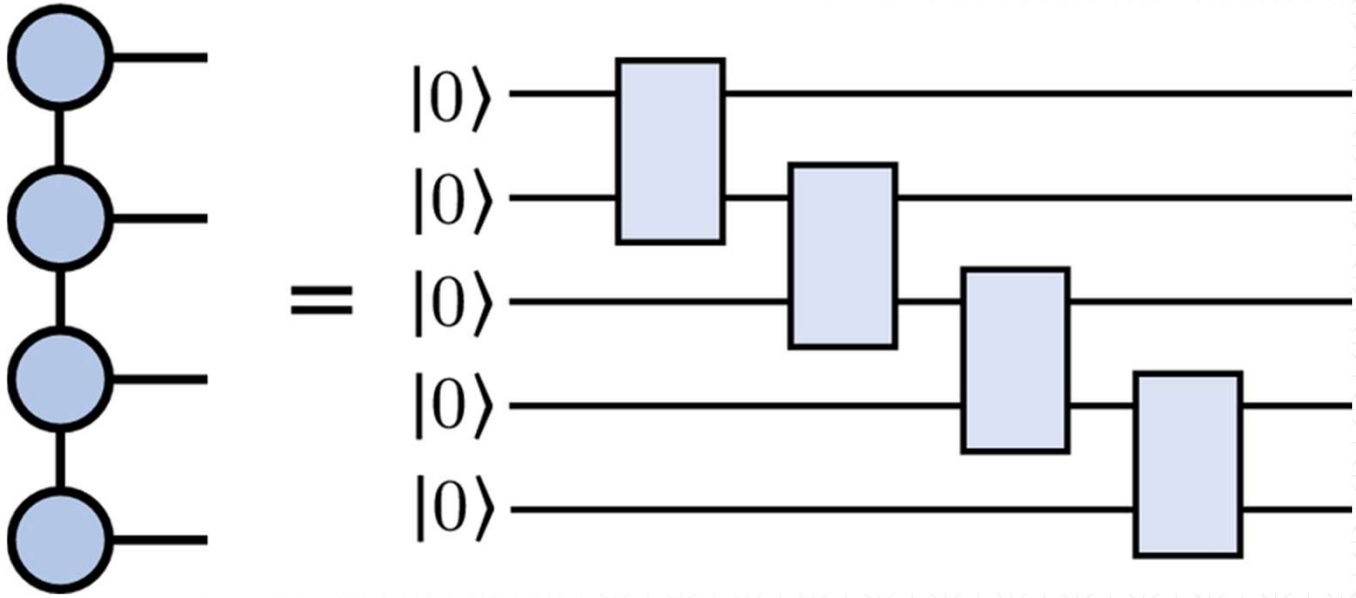


MERA



A Brief Introduction into Tensors

You can also perform operations:



A Brief Introduction into Tensors

$$\mathrm{T}_{i_1,i_2,\ldots,i_n} = A^{i_1}_{\alpha_1} A^{i_2}_{\alpha_1,\alpha_2} \ldots A^{i_n}_{\alpha_{n-1},\alpha_n}$$

Definitions:

$$\chi = \max(\alpha_i)$$

“Bond Dimension”, encodes correlations

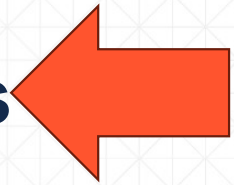
Applications

Tensor Networks have applications in:

1. Quantum Many-Body Simulations
2. Machine Learning
3. Quantum Algorithms (QML, Tomography, etc.)

An Ambitious Agenda

1. ~~The themes of AdS/CFT~~
2. ~~What are Tensor Networks?~~
3. **Holographic Tensor Networks**
4. Holographic Quantum Algorithms



Tensor Networks and Holography

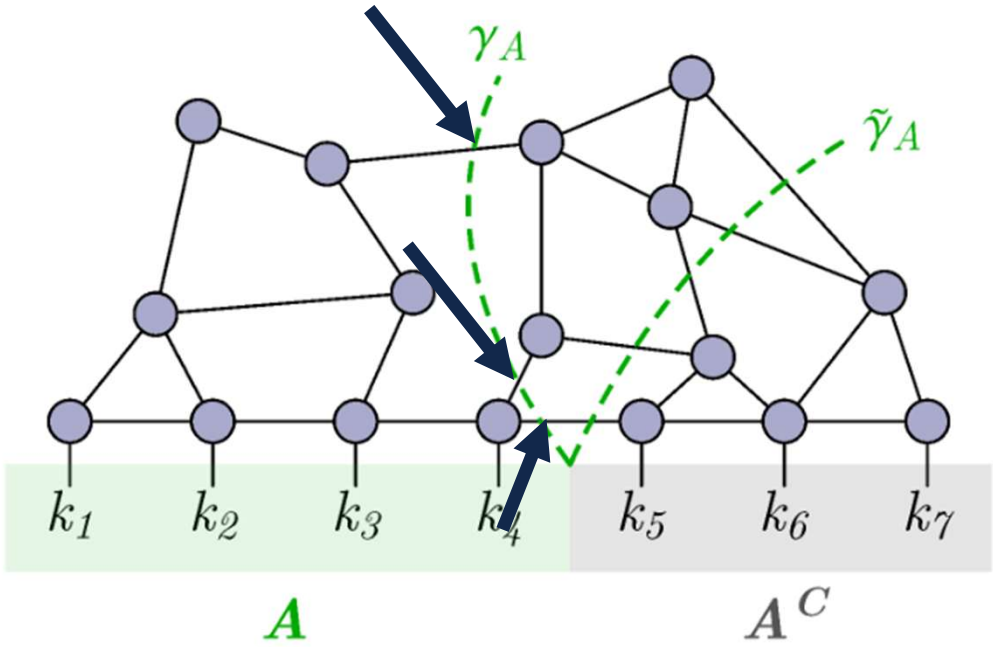
Tensor Networks have an Entanglement Entropy Bound:

$$S(A) \leq (\#\ \text{bonds cut}) \sum \chi$$

What does this mean?

Tensor Networks and Holography

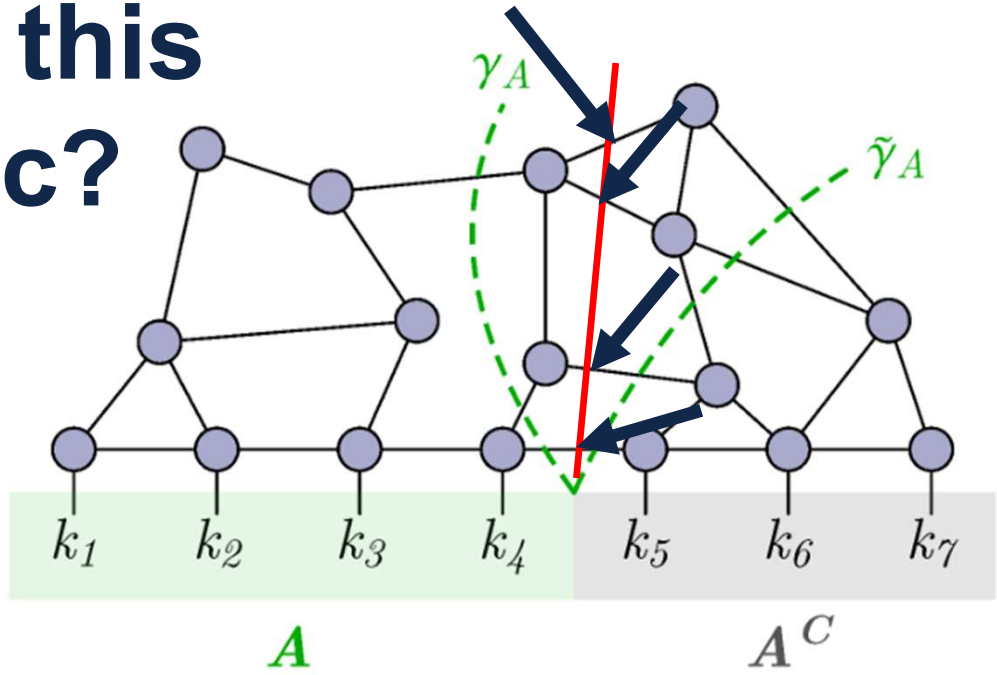
$$S(A) \leq (\#\ \text{bonds cut}) \sum \chi$$



Tensor Networks and Holography

$$S(A) \leq (\#\ \text{bonds cut}) \sum \chi$$

But how is this holographic?

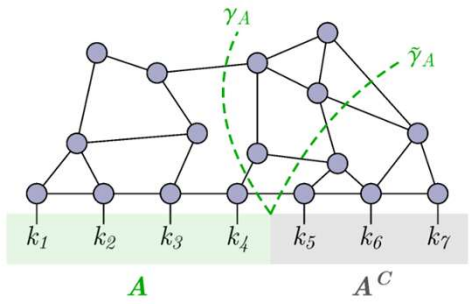


What if?

Lives in physical indices

Lives in virtual indices

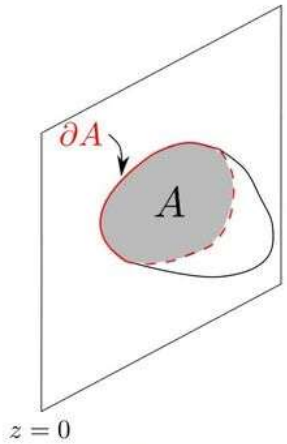
$$S(A) \propto (\#\ \text{bonds cut}) \sum \chi$$



Then it looks like:

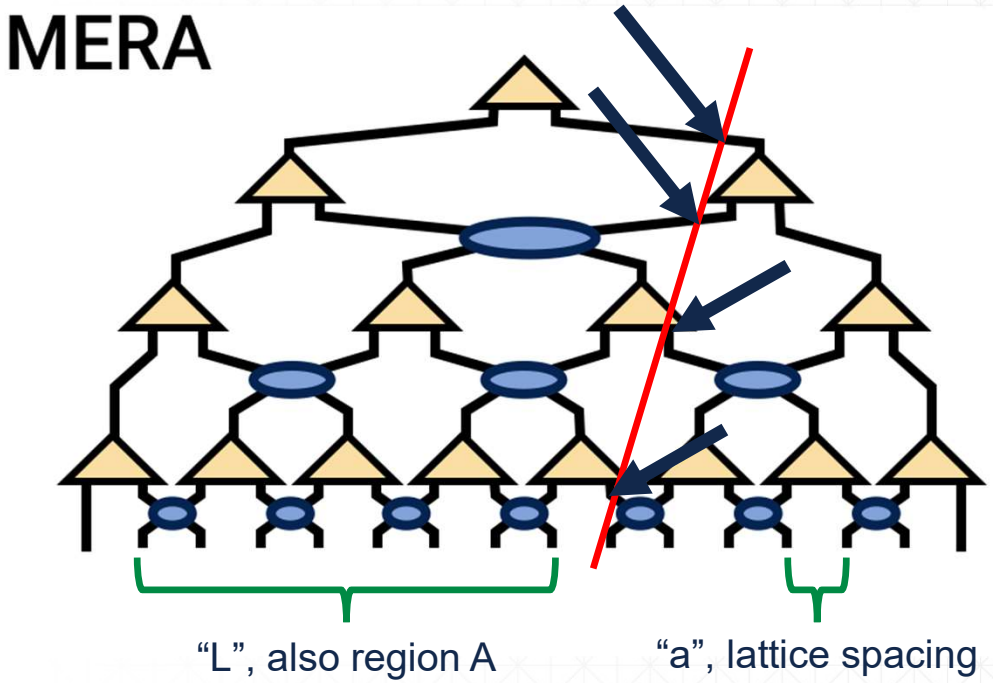
Entanglement Entropy of A ***in CFT***

$\propto$ Area of Curve (γ) ***in AdS***

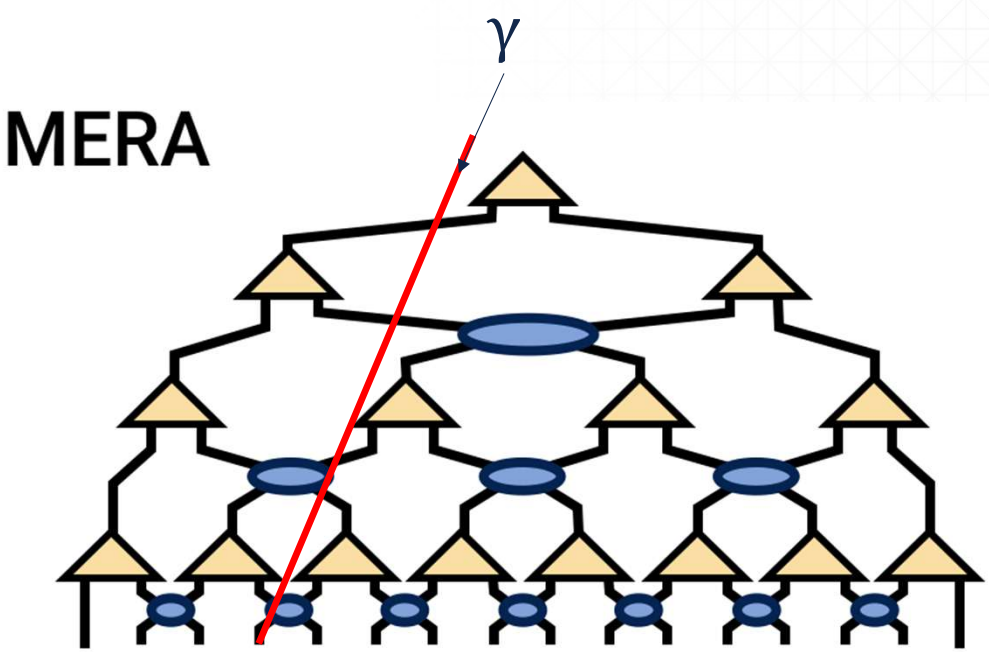


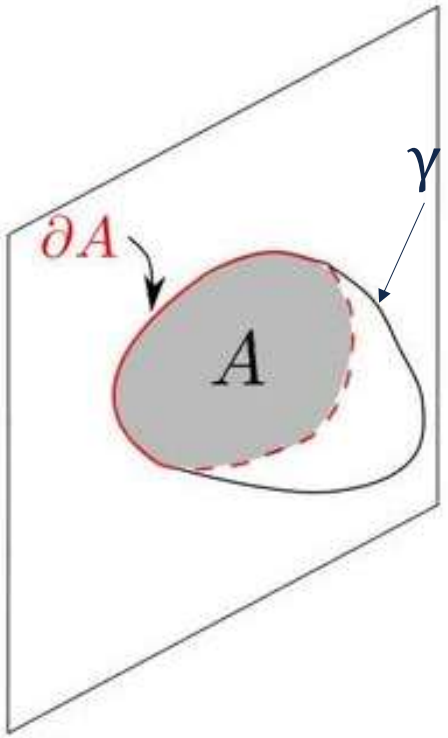
Introducing MERA

$S(A) \propto \log(L/a)$ **"Area Scaling Law"**



Connection between MERA and AdS/CFT





Both γ curves live in the "Virtual Degrees of Freedom" space

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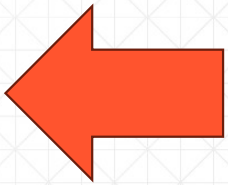


A Tour of Holographic Algorithms

Split up discussion into

1. MERA Applications
2. Holographic Error Correction
3. My Experience

MERA Applications

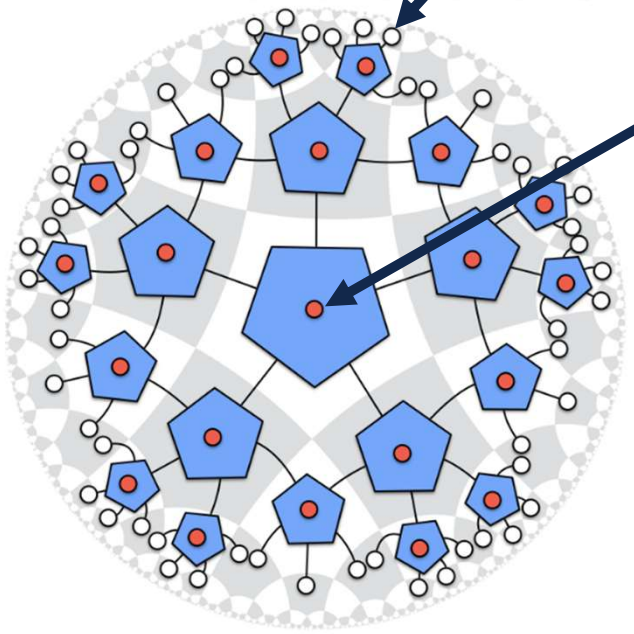
1. “Scale-invariant” Quantum Systems

$$S(A) \propto \log(L/a)$$

2. Extracting CFT information in quantum systems in the middle of a phase transition (critical)
3. Inspired ML architectures that process local and long range correlations (NLP, image processing)

Holographic Error Correction (HaPPY Code)

$V: H_{virtual} \rightarrow H_{Physical}$

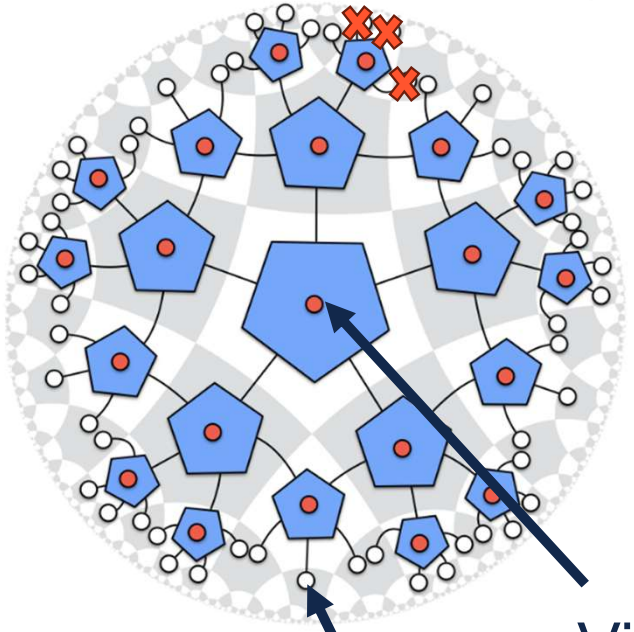


Holographic Error Correction (HaPPY Code)

$V: H_{virtual} \rightarrow H_{Physical}$

Remove some physical indices

Can we still find a O_P?



Goal: Given operator O_V

(1) Acting on virtual indices, but we want

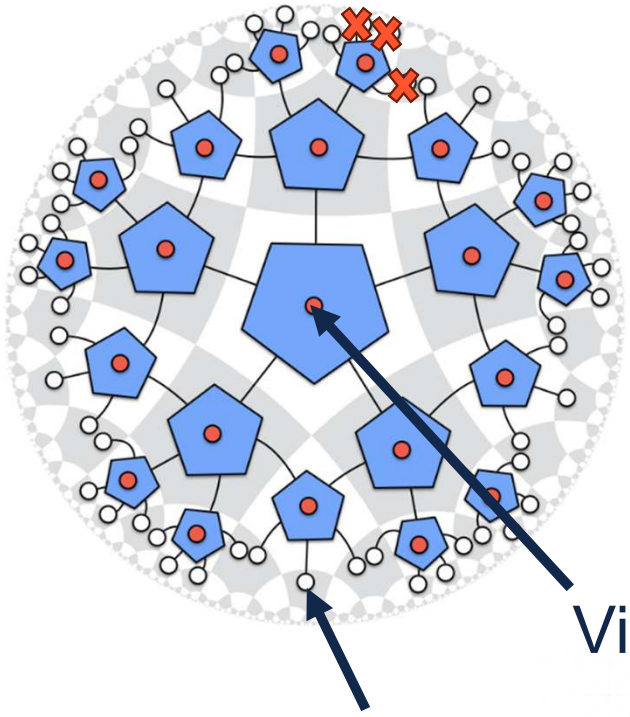
(2) Act on physical indices

That is, find O_P **(not unique)** such that $O_P = V^\dagger O_V V$

Virtual Indices

Physical Indices

Holographic Error Correction (HaPPY Code)



Goal: Given operator O_V
(1) Acting on virtual indices, but we want
(2) Act on physical indices

That is, find O_P **(not unique)** such that
$O_P = V^\dagger O_V V$

Solution: Combinatorics Algorithm

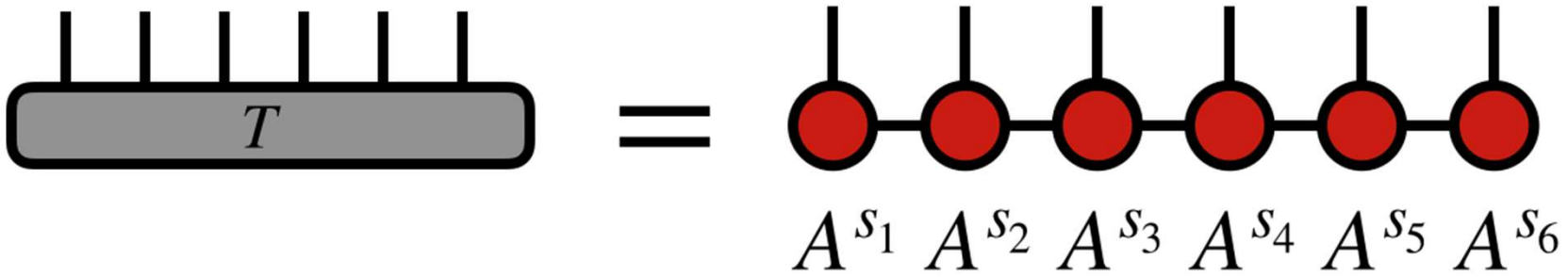
Idea: equivalent boundary and bulk description!

Pastawski et al., "Holographic quantum error-correcting codes: Toy models for the bulk/boundary Correspondence"

My experience @ Johns Hopkins Applied Physics Laboratory

Goal: Develop "holographic" generative QML algorithm

1.) Defined mapping $V: H_{\text{n qubits}} \rightarrow H_{\text{2 qubits}}$ for Matrix Product States

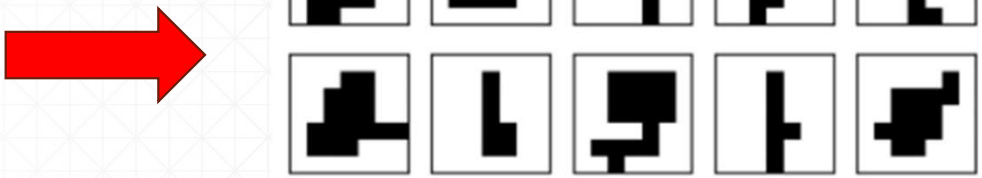


Only needed two qubits to "sequentially" prepare a n dimensional MPS

My experience @ Johns Hopkins Applied Physics Laboratory

2.) Derived sequential back propagation algorithm for 2 qubits

3.) Developed “holographic” generative QML algorithm & trained on NIST handwritten data



Now, I’ve pivoted to studying phase transitions in transformers for quantum compilation…

Closing remarks

1. Clearly new and emerging field
2. The connection between Tensor networks and AdS/CFT is still being strengthened
3. AdS/CFT is a surprisingly versatile idea

Using AdS/CFT for better Quantum Computing

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