

# Model Fitting Technique for Disordered Josephson Junction Arrays

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## INTRODUCTION

Josephson Junction Arrays (JJ-Arrays) are prevalent choices for constructing Superconducting Quantum Architecture, but their nanoscale nature makes it difficult to experimentally determine their geometric configuration. We propose a model-fitting technique to estimate the geometric configuration of these black-box systems from experimental data.

## EXPERIMENTAL DATA

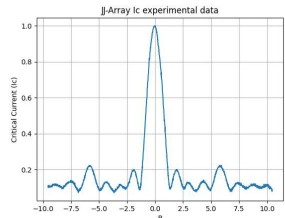


Fig 1. JJ-Array critical current experimental data

## TERMINOLOGY

### What is Gradient Descent?

- Calculates the gradient at a point and steps in the opposite direction for optimal decrease
- Commonly used in machine learning

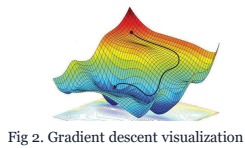


Fig 2. Gradient descent visualization

### What is a Cost & Cost Function?

- Cost is a scalar measurement of how different the true & computed values are
- Cost function maps optimization variables to a scalar representing cost

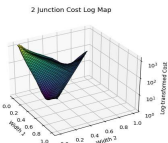


Fig 3. Two junction example cost function

### Cost Calculation = $\sum_{i=0}^n (x_i - y_i)^2$

- $x$  = Experimental  $I_c$  graph data point
- $y$  = Computed  $I_c$  graph data point
- $n$  = # of data points in an  $I_c$  graph

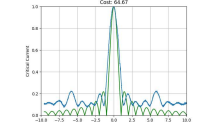


Fig 4. One junction example cost

## MODEL-FITTING ALGORITHM

**Full Name:** Momentum Gradient Descent with Random Perturbations in Reduced Parameter Space using Lattice Initial Conditions

### Momentum Gradient Descent...

- An extension of standard gradient descent
- Step vector "accumulates" momentum proportional to steepness of cost function
- Resilient to a cost function littered with minimas

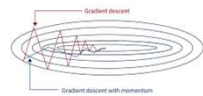


Fig 5. Momentum gradient descent visualization

### with Random Perturbations...

- Randomly generated vectors added to step vector
- Prevents trapping in local minima

### in Reduced Parameter Space...

- Use  $n$ -element array for  $n$ -junctions to describe centerline (C-L) of middle junctions
- Reduced the degrees of freedom  $2n - 2$  to  $n$  for an  $n$ -junction array for  $n \geq 2$

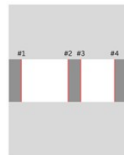


Fig 6. Degrees of freedom without fixed config (3-JJ's)

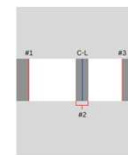


Fig 7. Degrees of freedom with fixed config [0, C-L, 1] (3-JJ's)

### using Lattice Initial Conditions

- Algorithm generates lattice of ~200 possible initial conditions for a given configuration (e.g. [0, 0.5, 1])

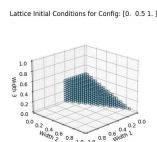


Fig 8. Example lattice initial conditions for [0, 0.5, 1]

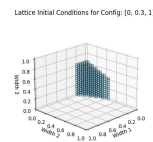


Fig 9. Example lattice initial conditions for [0, 0.3, 1]

## OPTIMIZING 2 JUNCTIONS

2 Junction Gradient Descent

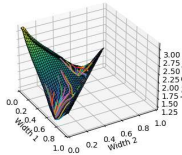


Fig 10. Two junction gradient descent (3D & heat map)

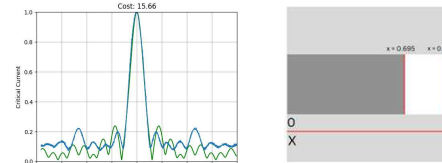


Fig 11. Two junction best fit & visualization  
Full device description: [0, 0.695, 0.964, 1]

## OPTIMIZING 3 JUNCTIONS

Gradient Vector Field for Config [0, 0.5, 1]

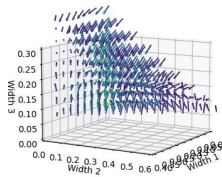


Fig 12. Example gradient vector field for valid widths associated with config [0, 0.5, 1]

Fig 13. Example cost versus gradient descent iterations using initial conditions from Fig. 8

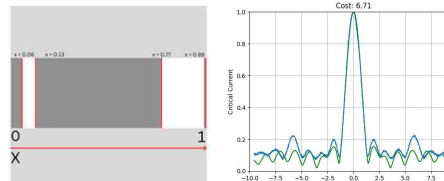
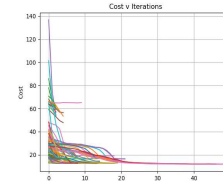


Fig 14. Three junction best fit & visualization  
Full device description: [0, 0.06, 0.13, 0.77, 0.99, 1]

## RESULTS

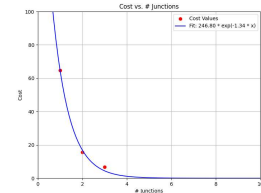


Fig 15. Best cost v # of junctions with best exponential fit

The best fit exponential equation was

$$f(x) = 246.8e^{-1.34x}$$

Therefore, given some cost threshold  $\tau$ , we can calculate the # of junctions needed to fulfill this

$$246.8e^{-1.34x} < \tau$$

$$x > \lceil -\frac{1}{1.34} \ln\left(\frac{\tau}{246.8}\right) \rceil$$

Letting  $\tau = 0.1$ , including 6 junctions in the device achieves a sufficient fit according to the model.

## FUTURE WORK

- Optimize the gradient descent and lattice generation algorithms for better time efficiency
- Validate best fit exponential model by optimizing 4, 5, & 6 junctions to exp data
- Package this as a public tool for researchers to use to estimate configuration of experimentally fabricated JJ-Arrays

## ACKNOWLEDGEMENTS

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