

Analyzing Chaos and Symmetry in a random Linear Josephson Junction Array

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INTRODUCTION

While symmetric, identically small-width Josephson Junction Arrays are well-understood and analytically solved, general JJ-Arrays remain less understood. In this poster, we propose a model of a general JJ-Array, analyzing its symmetries and exploring how its critical current changes as a function of magnetic field when increasing the randomness of junction widths.

WHAT IS A JOSEPHSON JUNCTION?

Comprised of a Superconductor-Insulator-Superconductor System (Fig 1.), Josephson Junctions operate off the Josephson Effect where the Cooper Pairs can Quantum Tunnel across the Insulator.

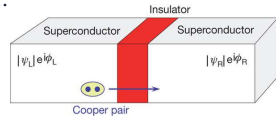


Fig. 1: Josephson Junction S-I-S System

WHAT IS A RANDOM LINEAR JOSEPHSON JUNCTION ARRAY?

Josephson Junctions are prevalent choices for constructing Superconducting Quantum Arrays with remarkable magnetic sensitivity. In particular, the 2-Junction Superconducting Quantum Interference Device (SQUID) (Fig. 2) stands out as a well-understood variant of the Linear Josephson Junction Array.

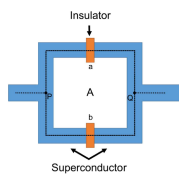


Fig 2. A Superconducting Quantum Interference Device



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SYMMETRY ANALYSIS

Equations:

$$I_s = J_c \sin(\Delta\phi)$$

Josephson Current-Phase Relationship

$$\Delta\phi(x) = \Delta\phi_0 + 2\pi bx$$

Meissner Phase Evolution

$\Delta\phi$ = phase difference across the Josephson Junction
 b = normalized magnetic flux w.r.t flux quantum
 x = spatial location of junction
 J_c = super-current density
 I_s = observed super-current

Results

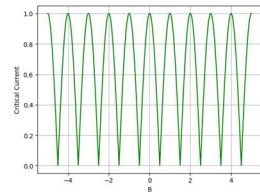


Fig 1. - 2 Identical Junction Array

Array = [0, 0.001, 0.999, 1]
 Junction #1 spans 0 to 0.001
 Junction #2 spans 0.999 to 1
 Mean Width = 0.001
 Standard Deviation = 0

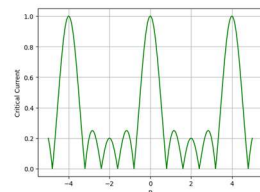


Fig 2. - 5 Identical Junction Array

Array = [0, 0.001, 0.25, 0.251, 0.5, 0.501, 0.75, 0.751, 0.999, 1]
 Mean Width = 0.001
 Standard Deviation = 0

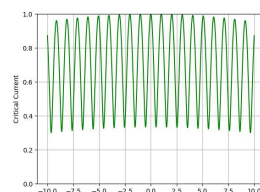


Fig 3. - 2 Different Junction Array

Array = [0, 0.02, 0.99, 1]
 Mean Width = 0.015
 Standard Deviation = 0.005

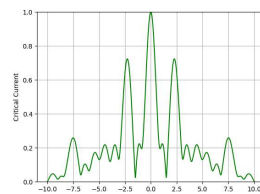


Fig 4. - 3 Random Junction Array

Array = [0, 0.2, 0.4, 0.6, 0.9, 1]
 Mean Width = 0.167
 Standard Deviation = 0.047

CHAOS ANALYSIS

Equation:

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2$$

Standard Deviation Formula

n = # of Josephson Junctions
 x_i = JJ-width
 μ = mean JJ-width
 σ = JJ-width standard deviation

Results – Incrementing σ by 0.01

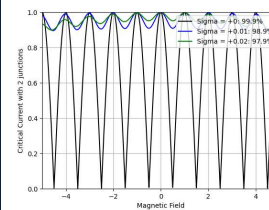


Fig 1. - 2 Identical Junction Array

Array #1 = [0, 0.001, 0.999, 1]
 Width Standard Deviation = 0
 Array #2 = [0, 0.021, 0.999, 1]
 Width Standard Deviation = 0.01
 Array #3 = [0, 0.001, 0.959, 1]
 Width Standard Deviation = 0.02

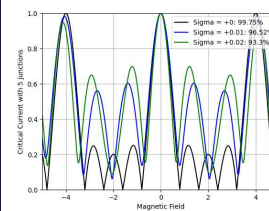


Fig 2. - 5 Identical Junction Array

Original Array = [0, 0.001, 0.2495, 0.2505, 0.4995, 0.5005, 0.7495, 0.7505, 0.999, 1]
 Respective Width Standard Deviations = 0, 0.01, 0.02

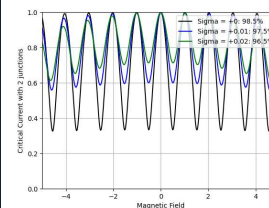


Fig 4. - 2 Different Junction Array

Original Array = [0, 0.02, 0.99, 1]
 Respective Width Standard Deviations = 0.005, 0.015, 0.025

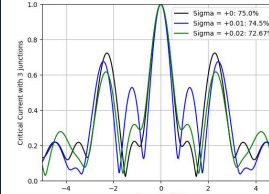


Fig 5. - 3 Random Junction Array

Original Array = [0, 0.2, 0.4, 0.6, 0.9, 1]
 Respective Width Standard Deviations = 0.047, 0.057, 0.067

CONCLUSIONS

Symmetry

We conclude that regardless of the Josephson Junction Array's configuration, a *guaranteed symmetry* exists across the critical current y-axis.

Chaos

We conclude that increasing the randomness of the Josephson Junction Array leads to a “node-lifting” effect. This represents the “zeros” of the critical current graph lifting as the junctions’ dimensions grow more random. As well, gradually increasing randomness in the junction dimensions typically results in gradual deviations in the critical current graphs, indicating non-chaotic behavior.

Future Discussion

We observe sensitivity in the JJ-Array correlating with the number of junctions and mean junction width. Specifically, JJ-arrays with fewer junctions and a smaller mean width appear to have a higher sensitivity to small perturbations in widths' standard deviation. This holds promise for future discussion.

REFERENCES

“Tinkham M Introduction to Superconductivity | PDF.” *Scribd*, www.scribd.com/document/666718455/59011645-Tinkham-M-Introduction-to-Superconductivity

ACKNOWLEDGEMENTS

I would like to thank my project co-contributor Prof. Alexey Bezryadin for his helpful inputs. I would also like to thank Harshvardhan Mantry for helpful discussions about the critical current model during its early development stages.



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